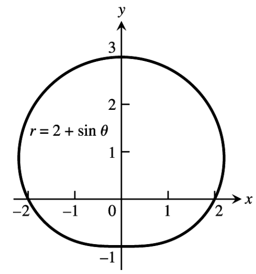
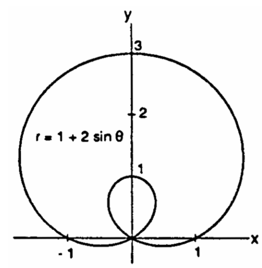
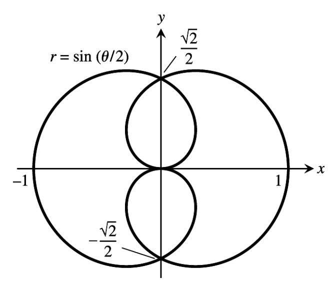
5. $2+\sin(-\theta)=2-\sin\theta \neq \mathrm{r}$ and $2+\sin(\pi-\theta)$
$=2+\sin\theta \neq -\mathrm{r} \Rightarrow$ not symmetric about the x-axis;
$2+\sin(\pi-\theta)=2+\sin\theta=\mathrm{r} \Rightarrow$ symmetric about the y-axis; therefore not symmetric about the origin



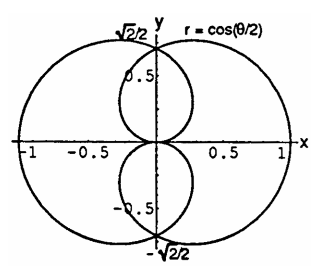
6. $1+2\sin(-\theta)=1-2\sin\theta \neq \mathrm{r}$ and $1+2\sin(\pi-\theta)$
$=1+2\sin\theta \neq -\mathrm{r} \Rightarrow$ not symmetric about the x-axis;
$1+2\sin(\pi-\theta)=1+2\sin\theta=\mathrm{r} \Rightarrow$ symmetric about the y-axis; therefore not symmetric about the origin



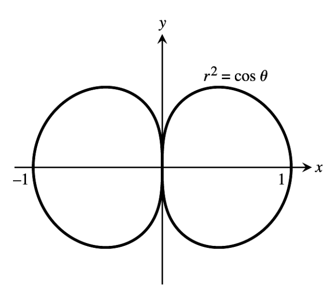
7. $\sin\left(-\frac{\theta}{2}\right)=-\sin\left(\frac{\theta}{2}\right)=-\mathrm{r} \Rightarrow$ symmetric about the y-axis;
$\sin\left(\frac{2\pi-\theta}{2}\right)=\sin\left(\frac{\theta}{2}\right)$, so the graph <u>is</u> symmetric about the x-axis, and hence the origin.



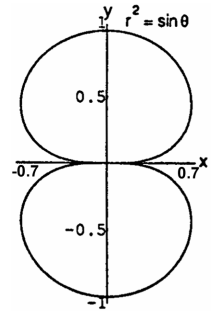
8. $\cos\left(-\frac{\theta}{2}\right)=\cos\left(\frac{\theta}{2}\right)=\mathrm{r} \Rightarrow$ symmetric about the x-axis;
$\cos\left(\frac{2\pi-\theta}{2}\right)=\cos\left(\frac{\theta}{2}\right)$, so the graph <u>is</u> symmetric about the y-axis, and hence the origin.



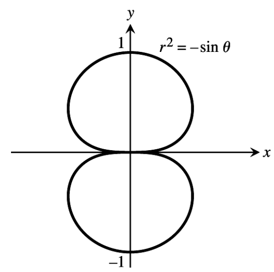
9. $\cos(-\theta)=\cos\theta=\mathrm{r}^2 \Rightarrow (\mathrm{r},-\theta)$ and $(-\mathrm{r},-\theta)$ are on the graph when (r,θ) is on the graph $\Rightarrow$ symmetric about the x-axis and the y-axis; therefore symmetric about the origin



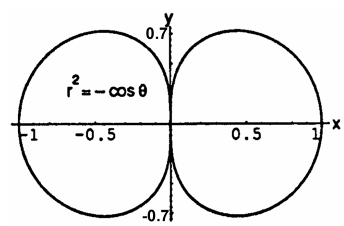
10. $\sin(\pi - \theta) = \sin\theta = r^2 \Rightarrow (r, \pi - \theta)$ and $(-r, \pi - \theta)$ are on the graph when (r, θ) is on the graph $\Rightarrow$ symmetric about the y-axis and the x-axis; therefore symmetric about the origin



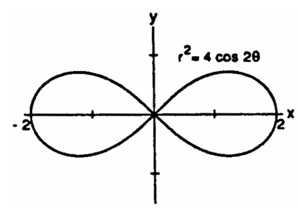
11. $-\sin(\pi - \theta) = -\sin\theta = r^2 \Rightarrow (r, \pi - \theta)$ and $(-r, \pi - \theta)$ are on the graph when (r, θ) is on the graph $\Rightarrow$ symmetric about the y-axis and the x-axis; therefore symmetric about the origin



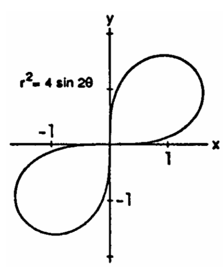
12. $-\cos(-\theta) = -\cos\theta = r^2 \Rightarrow (r, -\theta)$ and $(-r, -\theta)$ are on the graph when (r, θ) is on the graph $\Rightarrow$ symmetric about the x-axis and the y-axis; therefore symmetric about the origin



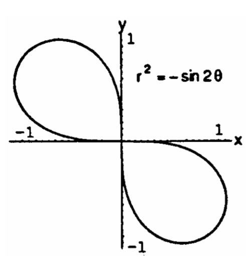
13. Since $(\pm r, -\theta)$ are on the graph when (r, θ) is on the graph $\left((\pm r)^2 = 4\cos 2(-\theta) \Rightarrow r^2 = 4\cos 2\theta\right)$, the graph is symmetric about the x-axis and the y-axis $\Rightarrow$ the graph is symmetric about the origin



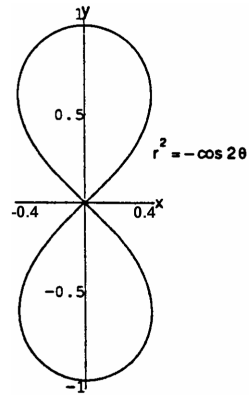
14. Since (r, θ) on the graph $\Rightarrow (-r, \theta)$ is on the graph $\left((\pm r)^2 = 4\sin 2\theta \Rightarrow r^2 = 4\sin 2\theta\right)$, the graph is symmetric about the origin. But $4\sin 2(-\theta) = -4\sin 2\theta$ $\neq r^2$ and $4\sin 2(\pi - \theta) = 4\sin(2\pi - 2\theta) = 4\sin(-2\theta)$ $= -4\sin 2\theta \neq r^2 \Rightarrow$ the graph is not symmetric about the x-axis; therefore the graph is not symmetric about the y-axis



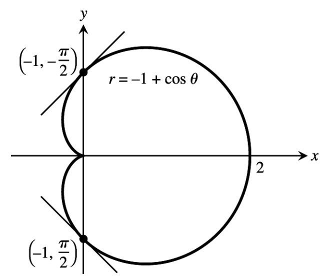
15. Since (r, θ) on the graph $\Rightarrow (-r, \theta)$ is on the graph $\left((\pm r)^2 = -\sin 2\theta \Rightarrow r^2 = -\sin 2\theta\right)$, the graph is symmetric about the origin. But $-\sin 2(-\theta) = -(-\sin 2\theta)$ $\sin 2\theta \neq r^2$ and $-\sin 2(\pi - \theta) = -\sin(2\pi - 2\theta)$ $= -\sin(-2\theta) = -(-\sin 2\theta) = \sin 2\theta \neq r^2 \Rightarrow$ the graph is not symmetric about the x-axis; therefore the graph is not symmetric about the y-axis



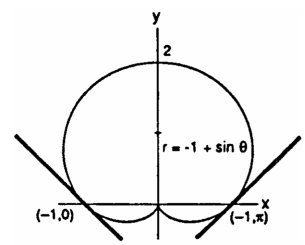
16. Since$(\pm r, -\theta)$ are on the graph when (r, θ) is on the graph $\left((\pm r)^2 = -\cos 2(-\theta) \Rightarrow r^2 = -\cos 2\theta\right)$, the graph is symmetric about the x-axis and the y-axis $\Rightarrow$ the graph is symmetric about the origin.



17. $\theta = \frac{\pi}{2} \Rightarrow r = -1 \Rightarrow \left(-1, \frac{\pi}{2}\right)$, and $\theta = -\frac{\pi}{2} \Rightarrow r = -1 \Rightarrow \left(-1, -\frac{\pi}{2}\right)$; $r' = \frac{dr}{d\theta} = -\sin\theta$; Slope $= \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$ $= \frac{-\sin^2\theta + r\cos\theta}{-\sin\theta\cos\theta - r\sin\theta} \Rightarrow$ Slope at $\left(-1, \frac{\pi}{2}\right)$ is $\frac{-\sin^2\left(\frac{\pi}{2}\right) + (-1)\cos\frac{\pi}{2}}{-\sin\frac{\pi}{2}\cos\frac{\pi}{2} - (-1)\sin\frac{\pi}{2}} = -1$; Slope at $\left(-1, -\frac{\pi}{2}\right)$ is $\frac{-\sin^2\left(-\frac{\pi}{2}\right) + (-1)\cos\left(-\frac{\pi}{2}\right)}{-\sin\left(-\frac{\pi}{2}\right)\cos\left(-\frac{\pi}{2}\right) - (-1)\sin\left(-\frac{\pi}{2}\right)} = 1$



18. $\theta = 0 \Rightarrow r = -1 \Rightarrow (-1, 0)$, and $\theta = \pi \Rightarrow r = -1 \Rightarrow (-1, \pi)$; $r' = \frac{dr}{d\theta} = \cos\theta$;
Slope $= \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{\cos\theta\sin\theta + r\cos\theta}{\cos\theta\cos\theta - r\sin\theta}$ $= \frac{\cos\theta\sin\theta + r\cos\theta}{\cos^2\theta - r\sin\theta} \Rightarrow$ Slope at $(-1, 0)$ is $\frac{\cos 0 \sin 0 + (-1)\cos 0}{\cos^2 0 - (-1)\sin 0}$ $= -1$; Slope at $(-1, \pi)$ is $\frac{\cos\pi\sin\pi + (-1)\cos\pi}{\cos^2\pi - (-1)\sin\pi} = 1$

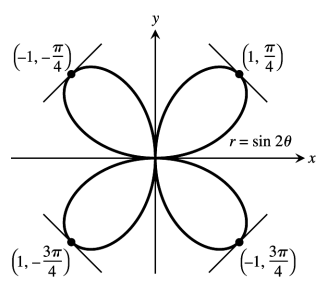


19. $\theta = \frac{\pi}{4} \Rightarrow r = 1 \Rightarrow \left(1, \frac{\pi}{4}\right)$; $\theta = -\frac{\pi}{4} \Rightarrow r = -1 \Rightarrow \left(-1, -\frac{\pi}{4}\right)$; $\theta = \frac{3\pi}{4} \Rightarrow r = -1 \Rightarrow \left(-1, \frac{3\pi}{4}\right)$; $\theta = -\frac{3\pi}{4} \Rightarrow r = 1 \Rightarrow \left(1, -\frac{3\pi}{4}\right)$;
$r' = \frac{dr}{d\theta} = 2\cos 2\theta$;
Slope $= \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{2\cos 2\theta\sin\theta + r\cos\theta}{2\cos 2\theta\cos\theta - r\sin\theta}$
$\Rightarrow$ Slope at $\left(1, \frac{\pi}{4}\right)$ is $\frac{2\cos\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{4}\right) + (1)\cos\left(\frac{\pi}{4}\right)}{2\cos\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{4}\right) - (1)\sin\left(\frac{\pi}{4}\right)} = -1$;

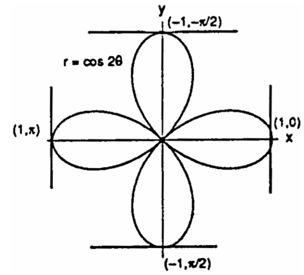
Slope at $\left(-1, -\frac{\pi}{4}\right)$ is $\frac{2\cos\left(-\frac{\pi}{2}\right)\sin\left(-\frac{\pi}{4}\right) + (-1)\cos\left(-\frac{\pi}{4}\right)}{2\cos\left(-\frac{\pi}{2}\right)\cos\left(-\frac{\pi}{4}\right) - (-1)\sin\left(-\frac{\pi}{4}\right)} = 1$;

Slope at $\left(-1, \frac{3\pi}{4}\right)$ is $\frac{2\cos\left(\frac{3\pi}{2}\right)\sin\left(\frac{3\pi}{4}\right) + (-1)\cos\left(\frac{3\pi}{4}\right)}{2\cos\left(\frac{3\pi}{2}\right)\cos\left(\frac{3\pi}{4}\right) - (-1)\sin\left(\frac{3\pi}{4}\right)} = 1$;

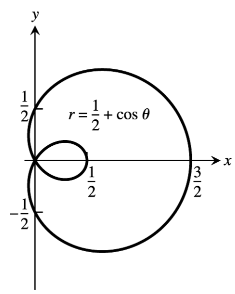
Slope at $\left(1, -\frac{3\pi}{4}\right)$ is $\frac{2\cos\left(-\frac{3\pi}{2}\right)\sin\left(-\frac{3\pi}{4}\right) + (1)\cos\left(-\frac{3\pi}{4}\right)}{2\cos\left(-\frac{3\pi}{2}\right)\cos\left(-\frac{3\pi}{4}\right) - (1)\sin\left(-\frac{3\pi}{4}\right)} = -1$



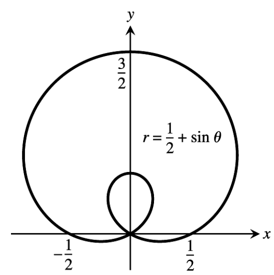
20. $\theta = 0 \Rightarrow r = 1 \Rightarrow (1, 0)$; $\theta = \frac{\pi}{2} \Rightarrow r = -1 \Rightarrow \left(-1, \frac{\pi}{2}\right)$;
$\theta = -\frac{\pi}{2} \Rightarrow r = -1 \Rightarrow \left(-1, -\frac{\pi}{2}\right)$; $\theta = \pi \Rightarrow r = 1$
$\Rightarrow (1, \pi)$; $r' = \frac{dr}{d\theta} = -2 \sin 2\theta$;
Slope $= \frac{r' \sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{-2\sin 2\theta \sin\theta + r\cos\theta}{-2\sin 2\theta\cos\theta - r\sin\theta}$
$\Rightarrow$ Slope at $(1, 0)$ is $\frac{-2\sin 0 \sin 0 + \cos 0}{-2\sin 0\cos 0 - \sin 0}$, which is undefined;
Slope at $\left(-1, \frac{\pi}{2}\right)$ is $\frac{-2\sin 2\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right) + (-1)\cos\left(\frac{\pi}{2}\right)}{-2\sin 2\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) - (-1)\sin\left(\frac{\pi}{2}\right)} = 0$;
Slope at $\left(-1, -\frac{\pi}{2}\right)$ is $\frac{-2\sin 2\left(-\frac{\pi}{2}\right)\sin\left(-\frac{\pi}{2}\right) + (-1)\cos\left(-\frac{\pi}{2}\right)}{-2\sin 2\left(-\frac{\pi}{2}\right)\cos\left(-\frac{\pi}{2}\right) - (-1)\sin\left(-\frac{\pi}{2}\right)} = 0$;
Slope at $(1, \pi)$ is $\frac{-2\sin 2\pi \sin\pi + \cos\pi}{-2\sin 2\pi\cos\pi - \sin\pi}$, which is undefined



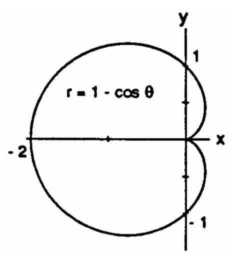
21. (a)



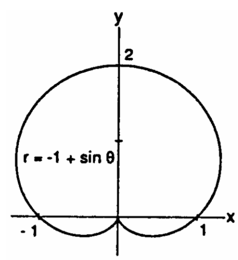
(b)



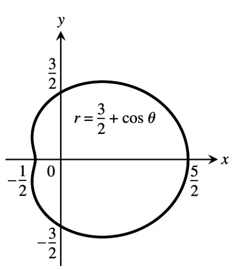
22. (a)



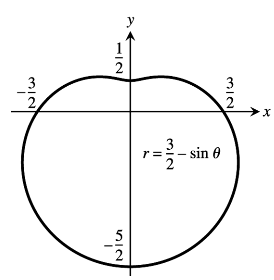
(b)



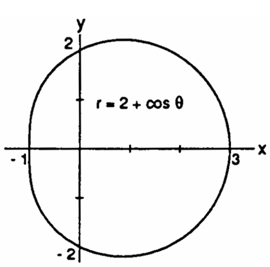
23. (a)



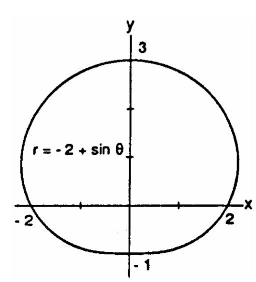
(b)



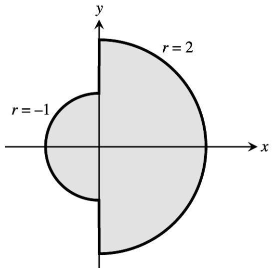
24. (a)



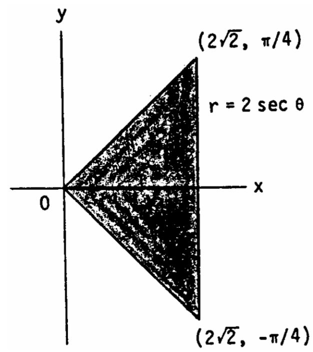
(b)



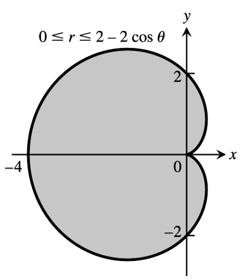
25.



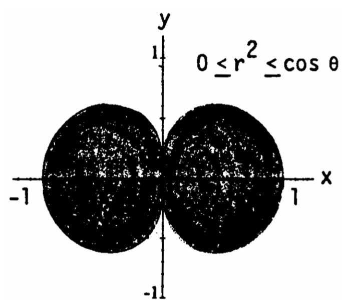
26. $r = 2 \sec \theta \;\Rightarrow\; r = \frac{2}{\cos \theta} \;\Rightarrow\; r \cos \theta = 2 \;\Rightarrow\; x = 2$



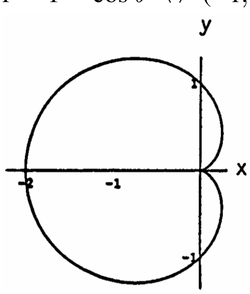
27.



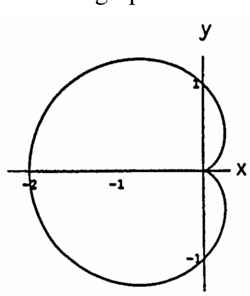
28.



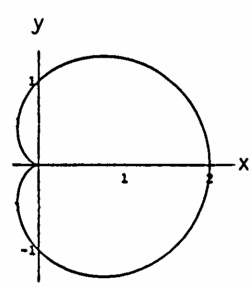
29. Note that (r, θ) and $(-r, \theta + \pi)$ describe the same point in the plane. Then $r = 1 - \cos \theta \;\Leftrightarrow\; -1 - \cos(\theta + \pi)$ $= -1 - (\cos \theta \cos \pi - \sin \theta \sin \pi) = -1 + \cos \theta = -(1 - \cos \theta) = -r$; therefore (r, θ) is on the graph of $r = 1 - \cos \theta \;\Leftrightarrow\; (-r, \theta + \pi)$ is on the graph of $r = -1 - \cos \theta \;\Rightarrow\;$ the answer is (a).



$r = 1 - \cos\theta$

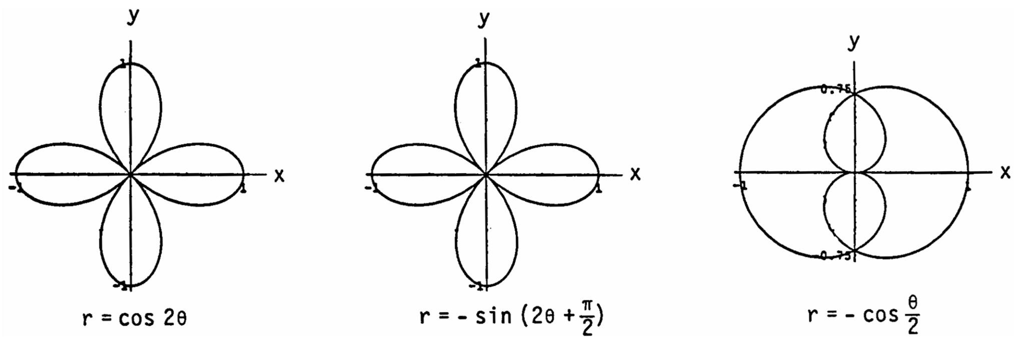


$r = -1 - \cos\theta$

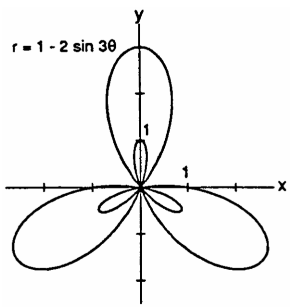


$r = 1 + \cos\theta$

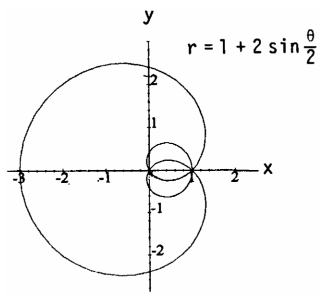
30. Note that (r, θ) and $(-r, \theta + \pi)$ describe the same point in the plane. Then $r = \cos 2\theta \Leftrightarrow -\sin\left(2(\theta+\pi)\right) + \frac{\pi}{2}\right)$ $= -\sin\left(2\theta + \frac{5\pi}{2}\right) = -\sin(2\theta)\cos\left(\frac{5\pi}{2}\right) - \cos(2\theta)\sin\left(\frac{5\pi}{2}\right) = -\cos 2\theta = -r$; therefore (r, θ) is on the graph of $r = -\sin\left(2\theta + \frac{\pi}{2}\right) \Rightarrow$ the answer is (a).



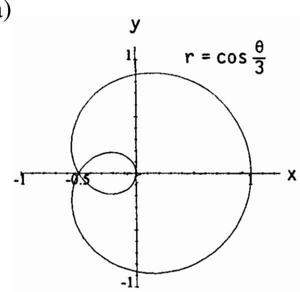
31.



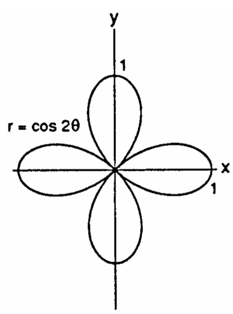
32.



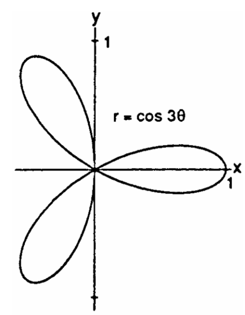
33. (a)



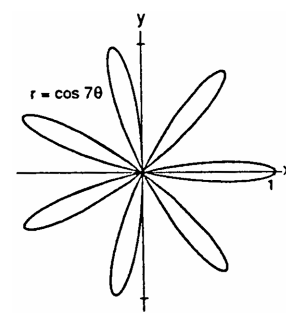
(b)



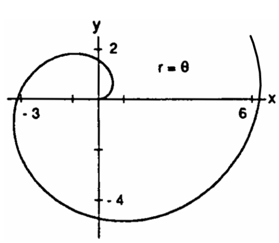
(c)



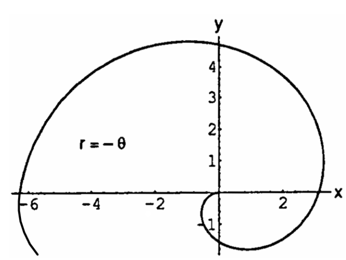
(d)



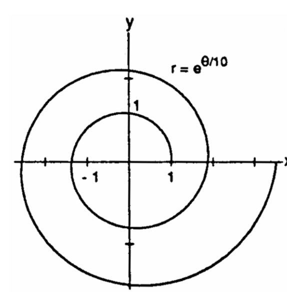
34. (a)



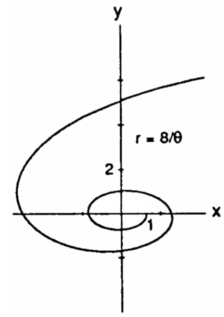
(b)



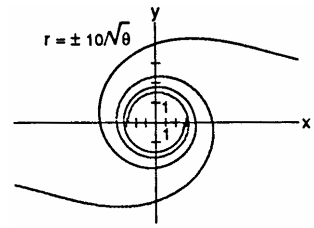
(c)



(d)



(e)



11.5 AREA AND LENGTHS IN POLAR COORDINATES

1. $A = \int_0^{\pi} \frac{1}{2}\theta^2\, d\theta = \left[\frac{1}{6}\theta^3\right]_0^{\pi} = \frac{\pi^3}{6}$

2. $A = \int_{\pi/4}^{\pi/2} \frac{1}{2}(2\sin\theta)^2\, d\theta = 2\int_{\pi/4}^{\pi/2} \sin^2\theta\, d\theta = 2\int_{\pi/4}^{\pi/2} \frac{1-\cos 2\theta}{2}\, d\theta = \int_{\pi/4}^{\pi/2} (1-\cos 2\theta) d\theta = \left[\theta - \frac{1}{2}\sin 2\theta\right]_{\pi/4}^{\pi/2}$
$= \left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi}{4} + \frac{1}{2}$

3. $A = \int_0^{2\pi} \frac{1}{2}(4 + 2\cos\theta)^2\, d\theta = \int_0^{2\pi} \frac{1}{2}(16 + 16\cos\theta + 4\cos^2\theta)\, d\theta = \int_0^{2\pi} \left[8 + 8\cos\theta + 2\left(\frac{1+\cos 2\theta}{2}\right)\right] d\theta$
$= \int_0^{2\pi} (9 + 8\cos\theta + \cos 2\theta)\, d\theta = \left[9\theta + 8\sin\theta + \frac{1}{2}\sin 2\theta\right]_0^{2\pi} = 18\pi$

4. $A = \int_0^{2\pi} \frac{1}{2}[a(1+\cos\theta)]^2\, d\theta = \int_0^{2\pi} \frac{1}{2}a^2(1 + 2\cos\theta + \cos^2\theta)\, d\theta = \frac{1}{2}a^2 \int_0^{2\pi} \left(1 + 2\cos\theta + \frac{1+\cos 2\theta}{2}\right) d\theta$
$= \frac{1}{2}a^2 \int_0^{2\pi} \left(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta\right) d\theta = \frac{1}{2}a^2\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^{2\pi} = \frac{3}{2}\pi a^2$

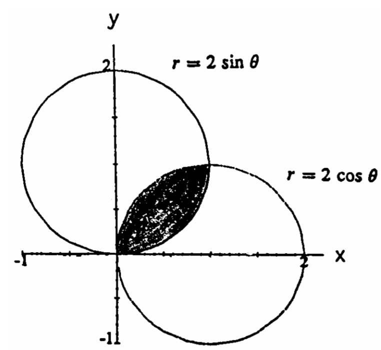
5. $A = 2\int_0^{\pi/4} \frac{1}{2}\cos^2 2\theta\, d\theta = \int_0^{\pi/4} \frac{1+\cos 4\theta}{2}\, d\theta = \frac{1}{2}\left[\theta + \frac{\sin 4\theta}{4}\right]_0^{\pi/4} = \frac{\pi}{8}$

6. $A = \int_{-\pi/6}^{\pi/6} \frac{1}{2}(\cos 3\theta)^2 d\theta = \frac{1}{2}\int_{-\pi/6}^{\pi/6} \cos^2 3\theta\, d\theta = \frac{1}{2}\int_{-\pi/6}^{\pi/6} \frac{1+\cos 6\theta}{2}\, d\theta = \frac{1}{4}\int_{-\pi/6}^{\pi/6} (1+\cos 6\theta)\, d\theta$
$= \frac{1}{4}\left[\theta + \frac{1}{6}\sin 6\theta\right]_{-\pi/6}^{\pi/6} = \frac{1}{4}\left(\frac{\pi}{6} + 0\right) - \frac{1}{4}\left(-\frac{\pi}{6} + 0\right) = \frac{\pi}{12}$

7. $A = \int_0^{\pi/2} \frac{1}{2}(4\sin 2\theta)\, d\theta = \int_0^{\pi/2} 2\sin 2\theta\, d\theta = [-\cos 2\theta]_0^{\pi/2} = 2$

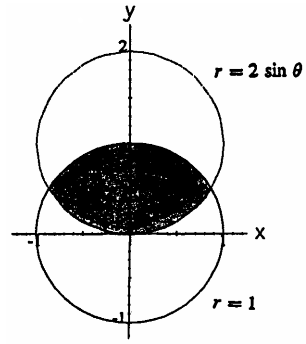
8. $A = (6)(2)\int_0^{\pi/6} \frac{1}{2}(2\sin 3\theta)\, d\theta = 12\int_0^{\pi/6} \sin 3\theta\, d\theta = 12\left[-\frac{\cos 3\theta}{3}\right]_0^{\pi/6} = 4$

9. $r = 2\cos\theta$ and $r = 2\sin\theta \Rightarrow 2\cos\theta = 2\sin\theta$
$\Rightarrow \cos\theta = \sin\theta \Rightarrow \theta = \frac{\pi}{4}$; therefore
$A = 2\int_0^{\pi/4} \frac{1}{2}(2\sin\theta)^2\, d\theta = \int_0^{\pi/4} 4\sin^2\theta\, d\theta$
$= \int_0^{\pi/4} 4\left(\frac{1-\cos 2\theta}{2}\right) d\theta = \int_0^{\pi/4} (2 - 2\cos 2\theta)\, d\theta$
$= [2\theta - \sin 2\theta]_0^{\pi/4} = \frac{\pi}{2} - 1$



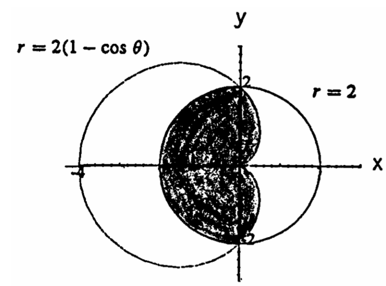
10. r $= 1$ and r $= 2 \sin \theta \;\Rightarrow\; 2 \sin \theta = 1 \;\Rightarrow\; \sin \theta = \frac{1}{2}$
$\Rightarrow\; \theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$; therefore

$$\mathrm{A} = \pi(1)^2 - \int_{\pi/6}^{5\pi/6} \tfrac{1}{2}\left[(2 \sin \theta)^2 - 1^2\right] d\theta$$
$$= \pi - \int_{\pi/6}^{5\pi/6} \left(2 \sin^2 \theta - \tfrac{1}{2}\right) d\theta$$
$$= \pi - \int_{\pi/6}^{5\pi/6} \left(1 - \cos 2\theta - \tfrac{1}{2}\right) d\theta$$
$$= \pi - \int_{\pi/6}^{5\pi/6} \left(\tfrac{1}{2} - \cos 2\theta\right) d\theta = \pi - \left[\tfrac{1}{2}\theta - \tfrac{\sin 2\theta}{2}\right]_{\pi/6}^{5\pi/6}$$
$$= \pi - \left(\tfrac{5\pi}{12} - \tfrac{1}{2} \sin \tfrac{5\pi}{3}\right) + \left(\tfrac{\pi}{12} - \tfrac{1}{2} \sin \tfrac{\pi}{3}\right) = \tfrac{4\pi - 3\sqrt{3}}{6}$$



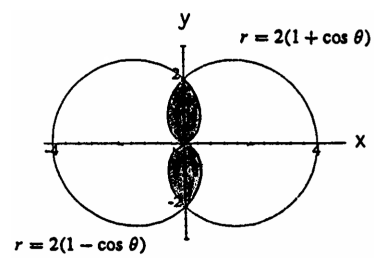
11. r $= 2$ and r $= 2(1 - \cos \theta) \;\Rightarrow\; 2 = 2(1 - \cos \theta)$
$\Rightarrow\; \cos \theta = 0 \;\Rightarrow\; \theta = \pm \frac{\pi}{2}$; therefore

$$\mathrm{A} = 2 \int_0^{\pi/2} \tfrac{1}{2}\left[2(1 - \cos \theta)\right]^2 d\theta + \tfrac{1}{2}\text{area of the circle}$$
$$= \int_0^{\pi/2} 4\left(1 - 2 \cos \theta + \cos^2 \theta\right) d\theta + \left(\tfrac{1}{2}\pi\right)(2)^2$$
$$= \int_0^{\pi/2} 4\left(1 - 2 \cos \theta + \tfrac{1 + \cos 2\theta}{2}\right) d\theta + 2\pi$$
$$= \int_0^{\pi/2} (4 - 8 \cos \theta + 2 + 2 \cos 2\theta)\, d\theta + 2\pi$$
$$= \left[6\theta - 8 \sin \theta + \sin 2\theta\right]_0^{\pi/2} + 2\pi = 5\pi - 8$$



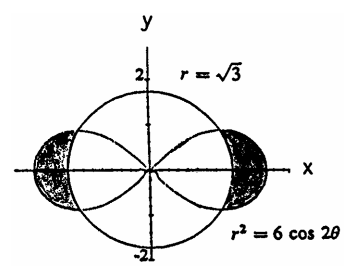
12. r $= 2(1 - \cos \theta)$ and r $= 2(1 + \cos \theta) \;\Rightarrow\; 1 - \cos \theta$
$= 1 + \cos \theta \;\Rightarrow\; \cos \theta = 0 \;\Rightarrow\; \theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$; the graph also
gives the point of intersection $(0, 0)$; therefore

$$\mathrm{A} = 2 \int_0^{\pi/2} \tfrac{1}{2}\left[2(1 - \cos \theta)\right]^2 d\theta + 2 \int_{\pi/2}^{\pi} \tfrac{1}{2}\left[2(1 + \cos \theta)\right]^2 d\theta$$
$$= \int_0^{\pi/2} 4(1 - 2\cos \theta + \cos^2 \theta) d\theta$$
$$+ \int_{\pi/2}^{\pi} 4\left(1 + 2 \cos \theta + \cos^2 \theta\right) d\theta$$
$$= \int_0^{\pi/2} 4\left(1 - 2 \cos \theta + \tfrac{1 + \cos 2\theta}{2}\right) d\theta + \int_{\pi/2}^{\pi} 4\left(1 + 2 \cos \theta + \tfrac{1 + \cos 2\theta}{2}\right) d\theta$$
$$= \int_0^{\pi/2} (6 - 8 \cos \theta + 2 \cos 2\theta)\, d\theta + \int_{\pi/2}^{\pi} (6 + 8 \cos \theta + 2 \cos 2\theta)\, d\theta$$
$$= \left[6\theta - 8 \sin \theta + \sin 2\theta\right]_0^{\pi/2} + \left[6\theta + 8 \sin \theta + \sin 2\theta\right]_{\pi/2}^{\pi} = 6\pi - 16$$

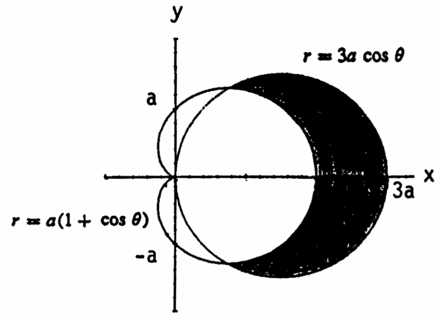


13. r $= \sqrt{3}$ and r$^2 = 6 \cos 2\theta \;\Rightarrow\; 3 = 6 \cos 2\theta \;\Rightarrow\; \cos 2\theta = \frac{1}{2}$
$\Rightarrow\; \theta = \frac{\pi}{6}$ (in the 1st quadrant); we use symmetry of the
graph to find the area, so

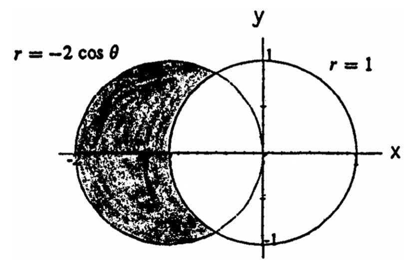
$$\mathrm{A} = 4 \int_0^{\pi/6} \left[\tfrac{1}{2}(6 \cos 2\theta) - \tfrac{1}{2}\left(\sqrt{3}\right)^2\right] d\theta$$
$$= 2 \int_0^{\pi/6} (6 \cos 2\theta - 3)\, d\theta = 2\left[3 \sin 2\theta - 3\theta\right]_0^{\pi/6}$$
$$= 3\sqrt{3} - \pi$$



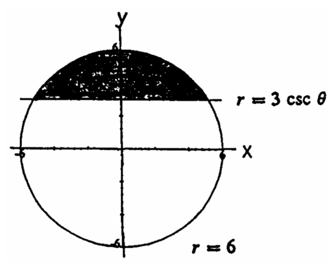
14. $r = 3a \cos\theta$ and $r = a(1+\cos\theta) \Rightarrow 3a\cos\theta = a(1+\cos\theta)$
$\Rightarrow\ 3\cos\theta = 1+\cos\theta\ \Rightarrow\ \cos\theta = \frac{1}{2}\ \Rightarrow\ \theta = \frac{\pi}{3}$ or $-\frac{\pi}{3}$;
the graph also gives the point of intersection $(0, 0)$; therefore
$A = 2\int_0^{\pi/3} \frac{1}{2}\left[(3a\cos\theta)^2 - a^2(1+\cos\theta)^2\right] d\theta$
$= \int_0^{\pi/3} (9a^2\cos^2\theta - a^2 - 2a^2\cos\theta - a^2\cos^2\theta)\, d\theta$
$= \int_0^{\pi/3} (8a^2\cos^2\theta - 2a^2\cos\theta - a^2)\, d\theta$
$= \int_0^{\pi/3} [4a^2(1+\cos 2\theta) - 2a^2\cos\theta - a^2]\, d\theta$
$= \int_0^{\pi/3} (3a^2 + 4a^2\cos 2\theta - 2a^2\cos\theta)\, d\theta$
$= \left[3a^2\theta + 2a^2\sin 2\theta - 2a^2\sin\theta\right]_0^{\pi/3} = \pi a^2 + 2a^2\left(\frac{1}{2}\right) - 2a^2\left(\frac{\sqrt{3}}{2}\right) = a^2\left(\pi + 1 - \sqrt{3}\right)$



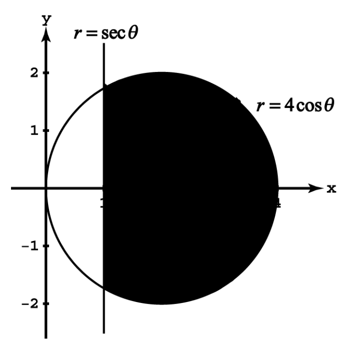
15. $r = 1$ and $r = -2\cos\theta\ \Rightarrow\ 1 = -2\cos\theta\ \Rightarrow\ \cos\theta = -\frac{1}{2}$
$\Rightarrow\ \theta = \frac{2\pi}{3}$ in quadrant II; therefore
$A = 2\int_{2\pi/3}^{\pi} \frac{1}{2}\left[(-2\cos\theta)^2 - 1^2\right] d\theta = \int_{2\pi/3}^{\pi} (4\cos^2\theta - 1)\, d\theta$
$= \int_{2\pi/3}^{\pi} [2(1+\cos 2\theta) - 1]\, d\theta = \int_{2\pi/3}^{\pi} (1 + 2\cos 2\theta)\, d\theta$
$= [\theta + \sin 2\theta]_{2\pi/3}^{\pi} = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$



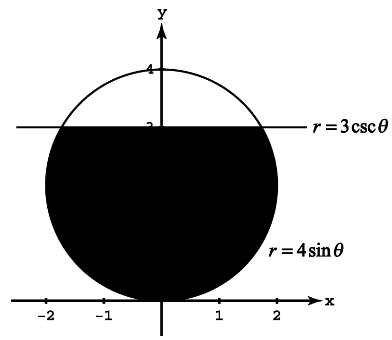
16. $r = 6$ and $r = 3\csc\theta\ \Rightarrow\ 6\sin\theta = 3\ \Rightarrow\ \sin\theta = \frac{1}{2}$
$\Rightarrow\ \theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$; therefore $A = \int_{\pi/6}^{5\pi/6} \frac{1}{2}(6^2 - 9\csc^2\theta)\, d\theta$
$= \int_{\pi/6}^{5\pi/6} \left(18 - \frac{9}{2}\csc^2\theta\right) d\theta = \left[18\theta + \frac{9}{2}\cot\theta\right]_{\pi/6}^{5\pi/6}$
$= \left(15\pi - \frac{9}{2}\sqrt{3}\right) - \left(3\pi + \frac{9}{2}\sqrt{3}\right) = 12\pi - 9\sqrt{3}$



17. $r = \sec\theta$ and $r = 4\cos\theta\ \Rightarrow 4\cos\theta = \sec\theta\ \Rightarrow\ \cos^2\theta = \frac{1}{4}$
$\Rightarrow\ \theta = \frac{\pi}{3},\ \frac{2\pi}{3},\ \frac{4\pi}{3}$, or $\frac{5\pi}{3}$; therefore
$A = 2\int_0^{\pi/3} \frac{1}{2}(16\cos^2\theta - \sec^2\theta)\, d\theta$
$= \int_0^{\pi/3} (8 + 8\cos 2\theta - \sec^2\theta)\, d\theta$
$= [8\theta + 4\sin 2\theta - \tan\theta]_0^{\pi/3}$
$= \left(\frac{8\pi}{3} + 2\sqrt{3} - \sqrt{3}\right) - (0 + 0 - 0) = \frac{8\pi}{3} + \sqrt{3}$

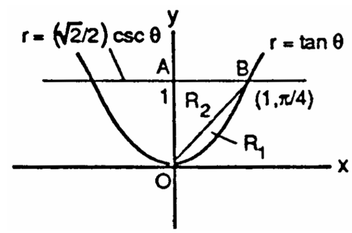


18. $r = 3\csc\theta$ and $r = 4\sin\theta \Rightarrow 4\sin\theta = 3\csc\theta \Rightarrow \sin^2\theta = \frac{3}{4}$
$\Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$, or $\frac{5\pi}{3}$; therefore



$$A = 4\pi - 2\int_{\pi/3}^{\pi/2} \frac{1}{2}\left(16\sin^2\theta - 9\csc^2\theta\right) d\theta$$
$$= 4\pi - \int_{\pi/3}^{\pi/2} \left(8 - 8\cos 2\theta - 9\csc^2\theta\right) d\theta$$
$$= 4\pi - \left[8\theta - 4\sin 2\theta + 9\cot\theta\right]_{\pi/3}^{\pi/2}$$
$$= 4\pi - \left[(4\pi - 0 + 0) - \left(\frac{8\pi}{3} - 2\sqrt{3} + 3\sqrt{3}\right)\right]$$
$$= \frac{8\pi}{3} + \sqrt{3}$$

19. (a) $r = \tan\theta$ and $r = \left(\frac{\sqrt{2}}{2}\right)\csc\theta \Rightarrow \tan\theta = \left(\frac{\sqrt{2}}{2}\right)\csc\theta$
$\Rightarrow \sin^2\theta = \left(\frac{\sqrt{2}}{2}\right)\cos\theta \Rightarrow 1 - \cos^2\theta = \left(\frac{\sqrt{2}}{2}\right)\cos\theta$
$\Rightarrow \cos^2\theta + \left(\frac{\sqrt{2}}{2}\right)\cos\theta - 1 = 0 \Rightarrow \cos\theta = -\sqrt{2}$ or
$\frac{\sqrt{2}}{2}$ (use the quadratic formula) $\Rightarrow \theta = \frac{\pi}{4}$ (the solution in the first quadrant); therefore the area of R_1 is



$A_1 = \int_0^{\pi/4} \frac{1}{2}\tan^2\theta\, d\theta = \frac{1}{2}\int_0^{\pi/4} (\sec^2\theta - 1)\, d\theta = \frac{1}{2}[\tan\theta - \theta]_0^{\pi/4} = \frac{1}{2}\left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) = \frac{1}{2} - \frac{\pi}{8}$; $AO = \left(\frac{\sqrt{2}}{2}\right)\csc\frac{\pi}{2}$
$= \frac{\sqrt{2}}{2}$ and $OB = \left(\frac{\sqrt{2}}{2}\right)\csc\frac{\pi}{4} = 1 \Rightarrow AB = \sqrt{1^2 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \Rightarrow$ the area of R_2 is $A_2 = \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{4}$;
therefore the area of the region shaded in the text is $2\left(\frac{1}{2} - \frac{\pi}{8} + \frac{1}{4}\right) = \frac{3}{2} - \frac{\pi}{4}$. Note: The area must be found this way since no common interval generates the region. For example, the interval $0 \le \theta \le \frac{\pi}{4}$ generates the arc OB of $r = \tan\theta$ but does not generate the segment AB of the line $r = \frac{\sqrt{2}}{2}\csc\theta$. Instead the interval generates the half-line from B to $+\infty$ on the line $r = \frac{\sqrt{2}}{2}\csc\theta$.

(b) $\lim_{\theta \to \pi/2^-} \tan\theta = \infty$ and the line $x = 1$ is $r = \sec\theta$ in polar coordinates; then $\lim_{\theta \to \pi/2^-} (\tan\theta - \sec\theta)$
$= \lim_{\theta \to \pi/2^-} \left(\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}\right) = \lim_{\theta \to \pi/2^-} \left(\frac{\sin\theta - 1}{\cos\theta}\right) = \lim_{\theta \to \pi/2^-} \left(\frac{\cos\theta}{-\sin\theta}\right) = 0 \Rightarrow r = \tan\theta$ approaches

$r = \sec\theta$ as $\theta \to \frac{\pi^-}{2} \Rightarrow r = \sec\theta$ (or $x = 1$) is a vertical asymptote of $r = \tan\theta$. Similarly, $r = -\sec\theta$ (or $x = -1$) is a vertical asymptote of $r = \tan\theta$.

20. It is not because the circle is generated twice from $\theta = 0$ to 2π. The area of the cardioid is
$A = 2\int_0^{\pi} \frac{1}{2}(\cos\theta + 1)^2\, d\theta = \int_0^{\pi} (\cos^2\theta + 2\cos\theta + 1)\, d\theta = \int_0^{\pi} \left(\frac{1 + \cos 2\theta}{2} + 2\cos\theta + 1\right) d\theta$
$= \left[\frac{3\theta}{2} + \frac{\sin 2\theta}{4} + 2\sin\theta\right]_0^{\pi} = \frac{3\pi}{2}$. The area of the circle is $A = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4} \Rightarrow$ the area requested is actually $\frac{3\pi}{2} - \frac{\pi}{4} = \frac{5\pi}{4}$

21. $r = \theta^2, 0 \le \theta \le \sqrt{5} \Rightarrow \frac{dr}{d\theta} = 2\theta$; therefore Length $= \int_0^{\sqrt{5}} \sqrt{(\theta^2)^2 + (2\theta)^2}\, d\theta = \int_0^{\sqrt{5}} \sqrt{\theta^4 + 4\theta^2}\, d\theta$
$= \int_0^{\sqrt{5}} |\theta|\sqrt{\theta^2 + 4}\, d\theta =$ (since $\theta \ge 0$) $\int_0^{\sqrt{5}} \theta\sqrt{\theta^2 + 4}\, d\theta$; $\left[u = \theta^2 + 4 \Rightarrow \frac{1}{2}\, du = \theta\, d\theta; \theta = 0 \Rightarrow u = 4,\right.$
$\left.\theta = \sqrt{5} \Rightarrow u = 9\right] \to \int_4^9 \frac{1}{2}\sqrt{u}\, du = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_4^9 = \frac{19}{3}$

22. $r = \frac{e^\theta}{\sqrt{2}}, 0 \le \theta \le \pi \Rightarrow \frac{dr}{d\theta} = \frac{e^\theta}{\sqrt{2}}$; therefore Length $= \int_0^{\pi} \sqrt{\left(\frac{e^\theta}{\sqrt{2}}\right)^2 + \left(\frac{e^\theta}{\sqrt{2}}\right)^2}\, d\theta = \int_0^{\pi} \sqrt{2\left(\frac{e^{2\theta}}{2}\right)}\, d\theta$
$= \int_0^{\pi} e^\theta\, d\theta = \left[e^\theta\right]_0^{\pi} = e^\pi - 1$

23. $r = 1 + \cos\theta \Rightarrow \frac{dr}{d\theta} = -\sin\theta$; therefore Length $= \int_0^{2\pi} \sqrt{(1+\cos\theta)^2 + (-\sin\theta)^2}\, d\theta$
$= 2\int_0^{\pi} \sqrt{2 + 2\cos\theta}\, d\theta = 2\int_0^{\pi} \sqrt{\frac{4(1+\cos\theta)}{2}}\, d\theta = 4\int_0^{\pi} \sqrt{\frac{1+\cos\theta}{2}}\, d\theta = 4\int_0^{\pi} \cos\left(\frac{\theta}{2}\right) d\theta = 4\left[2\sin\frac{\theta}{2}\right]_0^{\pi} = 8$

24. $r = a\sin^2\frac{\theta}{2},\ 0 \le \theta \le \pi,\ a > 0 \Rightarrow \frac{dr}{d\theta} = a\sin\frac{\theta}{2}\cos\frac{\theta}{2}$; therefore Length $= \int_0^{\pi} \sqrt{\left(a\sin^2\frac{\theta}{2}\right)^2 + \left(a\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^2}\, d\theta$
$= \int_0^{\pi} \sqrt{a^2\sin^4\frac{\theta}{2} + a^2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}\, d\theta = \int_0^{\pi} a\left|\sin\frac{\theta}{2}\right| \sqrt{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}}\, d\theta =$ (since $0 \le \theta \le \pi$) $a\int_0^{\pi} \sin\left(\frac{\theta}{2}\right) d\theta$
$= \left[-2a\cos\frac{\theta}{2}\right]_0^{\pi} = 2a$

25. $r = \frac{6}{1+\cos\theta},\ 0 \le \theta \le \frac{\pi}{2} \Rightarrow \frac{dr}{d\theta} = \frac{6\sin\theta}{(1+\cos\theta)^2}$; therefore Length $= \int_0^{\pi/2} \sqrt{\left(\frac{6}{1+\cos\theta}\right)^2 + \left(\frac{6\sin\theta}{(1+\cos\theta)^2}\right)^2}\, d\theta$
$= \int_0^{\pi/2} \sqrt{\frac{36}{(1+\cos\theta)^2} + \frac{36\sin^2\theta}{(1+\cos\theta)^4}}\, d\theta = 6\int_0^{\pi/2} \left|\frac{1}{1+\cos\theta}\right| \sqrt{1 + \frac{\sin^2\theta}{(1+\cos\theta)^2}}\, d\theta$
$=$ (since $\frac{1}{1+\cos\theta} > 0$ on $0 \le \theta \le \frac{\pi}{2}$) $6\int_0^{\pi/2} \left(\frac{1}{1+\cos\theta}\right) \sqrt{\frac{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}{(1+\cos\theta)^2}}\, d\theta$
$= 6\int_0^{\pi/2} \left(\frac{1}{1+\cos\theta}\right) \sqrt{\frac{2+2\cos\theta}{(1+\cos\theta)^2}}\, d\theta = 6\sqrt{2}\int_0^{\pi/2} \frac{d\theta}{(1+\cos\theta)^{3/2}} = 6\sqrt{2}\int_0^{\pi/2} \frac{d\theta}{\left(2\cos^2\frac{\theta}{2}\right)^{3/2}} = 3\int_0^{\pi/2} \left|\sec^3\frac{\theta}{2}\right| d\theta$
$= 3\int_0^{\pi/2} \sec^3\frac{\theta}{2}\, d\theta = 6\int_0^{\pi/4} \sec^3 u\, du =$ (use tables) $6\left(\left[\frac{\sec u \tan u}{2}\right]_0^{\pi/4} + \frac{1}{2}\int_0^{\pi/4} \sec u\, du\right)$
$= 6\left(\frac{1}{\sqrt{2}} + \left[\frac{1}{2}\ln|\sec u + \tan u|\right]_0^{\pi/4}\right) = 3\left[\sqrt{2} + \ln\left(1 + \sqrt{2}\right)\right]$

26. $r = \frac{2}{1-\cos\theta},\ \frac{\pi}{2} \le \theta \le \pi \Rightarrow \frac{dr}{d\theta} = \frac{-2\sin\theta}{(1-\cos\theta)^2}$; therefore Length $= \int_{\pi/2}^{\pi} \sqrt{\left(\frac{2}{1-\cos\theta}\right)^2 + \left(\frac{-2\sin\theta}{(1-\cos\theta)^2}\right)^2}\, d\theta$
$= \int_{\pi/2}^{\pi} \sqrt{\frac{4}{(1-\cos\theta)^2}\left(1 + \frac{\sin^2\theta}{(1-\cos\theta)^2}\right)}\, d\theta = \int_{\pi/2}^{\pi} \left|\frac{2}{1-\cos\theta}\right| \sqrt{\frac{(1-\cos\theta)^2 + \sin^2\theta}{(1-\cos\theta)^2}}\, d\theta$
$=$ (since $1 - \cos\theta \ge 0$ on $\frac{\pi}{2} \le \theta \le \pi$) $2\int_{\pi/2}^{\pi} \left(\frac{1}{1-\cos\theta}\right) \sqrt{\frac{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}{(1-\cos\theta)^2}}\, d\theta$
$= 2\int_{\pi/2}^{\pi} \left(\frac{1}{1-\cos\theta}\right) \sqrt{\frac{2-2\cos\theta}{(1-\cos\theta)^2}}\, d\theta = 2\sqrt{2}\int_{\pi/2}^{\pi} \frac{d\theta}{(1-\cos\theta)^{3/2}} = 2\sqrt{2}\int_{\pi/2}^{\pi} \frac{d\theta}{\left(2\sin^2\frac{\theta}{2}\right)^{3/2}} = \int_{\pi/2}^{\pi} \left|\csc^3\frac{\theta}{2}\right| d\theta$
$= \int_{\pi/2}^{\pi} \csc^3\left(\frac{\theta}{2}\right) d\theta =$ (since $\csc\frac{\theta}{2} \ge 0$ on $\frac{\pi}{2} \le \theta \le \pi$) $2\int_{\pi/4}^{\pi/2} \csc^3 u\, du =$ (use tables)
$2\left(\left[-\frac{\csc u \cot u}{2}\right]_{\pi/4}^{\pi/2} + \frac{1}{2}\int_{\pi/4}^{\pi/2} \csc u\, du\right) = 2\left(\frac{1}{\sqrt{2}} - \left[\frac{1}{2}\ln|\csc u + \cot u|\right]_{\pi/4}^{\pi/2}\right) = 2\left[\frac{1}{\sqrt{2}} + \frac{1}{2}\ln\left(\sqrt{2} + 1\right)\right]$
$= \sqrt{2} + \ln\left(1 + \sqrt{2}\right)$

27. $r = \cos^3\frac{\theta}{3} \Rightarrow \frac{dr}{d\theta} = -\sin\frac{\theta}{3}\cos^2\frac{\theta}{3}$; therefore Length $= \int_0^{\pi/4} \sqrt{\left(\cos^3\frac{\theta}{3}\right)^2 + \left(-\sin\frac{\theta}{3}\cos^2\frac{\theta}{3}\right)^2}\, d\theta$
$= \int_0^{\pi/4} \sqrt{\cos^6\left(\frac{\theta}{3}\right) + \sin^2\left(\frac{\theta}{3}\right)\cos^4\left(\frac{\theta}{3}\right)}\, d\theta = \int_0^{\pi/4} \left(\cos^2\frac{\theta}{3}\right) \sqrt{\cos^2\left(\frac{\theta}{3}\right) + \sin^2\left(\frac{\theta}{3}\right)}\, d\theta = \int_0^{\pi/4} \cos^2\left(\frac{\theta}{3}\right) d\theta$
$= \int_0^{\pi/4} \frac{1 + \cos\left(\frac{2\theta}{3}\right)}{2}\, d\theta = \frac{1}{2}\left[\theta + \frac{3}{2}\sin\frac{2\theta}{3}\right]_0^{\pi/4} = \frac{\pi}{8} + \frac{3}{8}$

28. $r = \sqrt{1 + \sin 2\theta},\ 0 \le \theta \le \pi\sqrt{2} \Rightarrow \frac{dr}{d\theta} = \frac{1}{2}(1 + \sin 2\theta)^{-1/2}(2\cos 2\theta) = (\cos 2\theta)(1 + \sin 2\theta)^{-1/2}$; therefore
Length $= \int_0^{\pi\sqrt{2}} \sqrt{(1 + \sin 2\theta) + \frac{\cos^2 2\theta}{(1 + \sin 2\theta)}}\, d\theta = \int_0^{\pi\sqrt{2}} \sqrt{\frac{1 + 2\sin 2\theta + \sin^2 2\theta + \cos^2 2\theta}{1 + \sin 2\theta}}\, d\theta$
$= \int_0^{\pi\sqrt{2}} \sqrt{\frac{2 + 2\sin 2\theta}{1 + \sin 2\theta}}\, d\theta = \int_0^{\pi\sqrt{2}} \sqrt{2}\, d\theta = \left[\sqrt{2}\,\theta\right]_0^{\pi\sqrt{2}} = 2\pi$

29. Let $r = f(\theta)$. Then $x = f(\theta)\cos\theta \Rightarrow \frac{dx}{d\theta} = f'(\theta)\cos\theta - f(\theta)\sin\theta \Rightarrow \left(\frac{dx}{d\theta}\right)^2 = [f'(\theta)\cos\theta - f(\theta)\sin\theta]^2$ $= [f'(\theta)]^2\cos^2\theta - 2f'(\theta)\,f(\theta)\sin\theta\cos\theta + [f(\theta)]^2\sin^2\theta$; $y = f(\theta)\sin\theta \Rightarrow \frac{dy}{d\theta} = f'(\theta)\sin\theta + f(\theta)\cos\theta$ $\Rightarrow \left(\frac{dy}{d\theta}\right)^2 = [f'(\theta)\sin\theta + f(\theta)\cos\theta]^2 = [f'(\theta)]^2\sin^2\theta + 2f'(\theta)f(\theta)\sin\theta\cos\theta + [f(\theta)]^2\cos^2\theta$. Therefore $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = [f'(\theta)]^2(\cos^2\theta + \sin^2\theta) + [f(\theta)]^2(\cos^2\theta + \sin^2\theta) = [f'(\theta)]^2 + [f(\theta)]^2 = r^2 + \left(\frac{dr}{d\theta}\right)^2$.
Thus, $L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\,d\theta = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$.

30. (a) $r = a \Rightarrow \frac{dr}{d\theta} = 0$; Length $= \int_0^{2\pi}\sqrt{a^2 + 0^2}\,d\theta = \int_0^{2\pi}|a|\,d\theta = [a\theta]_0^{2\pi} = 2\pi a$

(b) $r = a\cos\theta \Rightarrow \frac{dr}{d\theta} = -a\sin\theta$; Length $= \int_0^\pi \sqrt{(a\cos\theta)^2 + (-a\sin\theta)^2}\,d\theta = \int_0^\pi \sqrt{a^2(\cos^2\theta + \sin^2\theta)}\,d\theta$ $= \int_0^\pi |a|\,d\theta = [a\theta]_0^\pi = \pi a$

(c) $r = a\sin\theta \Rightarrow \frac{dr}{d\theta} = a\cos\theta$; Length $= \int_0^\pi \sqrt{(a\cos\theta)^2 + (a\sin\theta)^2}\,d\theta = \int_0^\pi \sqrt{a^2(\cos^2\theta + \sin^2\theta)}\,d\theta$ $= \int_0^\pi |a|\,d\theta = [a\theta]_0^\pi = \pi a$

31. (a) $r_{av} = \frac{1}{2\pi - 0}\int_0^{2\pi} a(1 - \cos\theta)\,d\theta = \frac{a}{2\pi}[\theta - \sin\theta]_0^{2\pi} = a$

(b) $r_{av} = \frac{1}{2\pi - 0}\int_0^{2\pi} a\,d\theta = \frac{1}{2\pi}[a\theta]_0^{2\pi} = a$

(c) $r_{av} = \frac{1}{\left(\frac{\pi}{2}\right) - \left(-\frac{\pi}{2}\right)}\int_{-\pi/2}^{\pi/2} a\cos\theta\,d\theta = \frac{1}{\pi}[a\sin\theta]_{-\pi/2}^{\pi/2} = \frac{2a}{\pi}$

32. $r = 2f(\theta)$, $\alpha \le \theta \le \beta \Rightarrow \frac{dr}{d\theta} = 2f'(\theta) \Rightarrow r^2 + \left(\frac{dr}{d\theta}\right)^2 = [2f(\theta)]^2 + [2f'(\theta)]^2 \Rightarrow$ Length $= \int_\alpha^\beta \sqrt{4[f(\theta)]^2 + 4[f'(\theta)]^2}\,d\theta$ $= 2\int_\alpha^\beta \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\,d\theta$ which is twice the length of the curve $r = f(\theta)$ for $\alpha \le \theta \le \beta$.

11.6 CONIC SECTIONS

1. $x = \frac{y^2}{8} \Rightarrow 4p = 8 \Rightarrow p = 2$; focus is $(2, 0)$, directrix is $x = -2$

2. $x = -\frac{y^2}{4} \Rightarrow 4p = 4 \Rightarrow p = 1$; focus is $(-1, 0)$, directrix is $x = 1$

3. $y = -\frac{x^2}{6} \Rightarrow 4p = 6 \Rightarrow p = \frac{3}{2}$; focus is $\left(0, -\frac{3}{2}\right)$, directrix is $y = \frac{3}{2}$

4. $y = \frac{x^2}{2} \Rightarrow 4p = 2 \Rightarrow p = \frac{1}{2}$; focus is $\left(0, \frac{1}{2}\right)$, directrix is $y = -\frac{1}{2}$

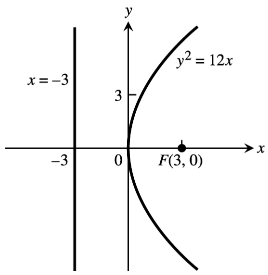
5. $\frac{x^2}{4} - \frac{y^2}{9} = 1 \Rightarrow c = \sqrt{4 + 9} = \sqrt{13} \Rightarrow$ foci are $\left(\pm\sqrt{13}, 0\right)$; vertices are $(\pm 2, 0)$; asymptotes are $y = \pm\frac{3}{2}x$

6. $\frac{x^2}{4} + \frac{y^2}{9} = 1 \Rightarrow c = \sqrt{9 - 4} = \sqrt{5} \Rightarrow$ foci are $\left(0, \pm\sqrt{5}\right)$; vertices are $(0, \pm 3)$

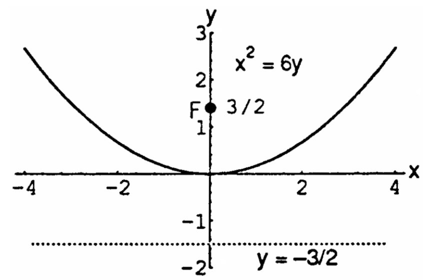
7. $\frac{x^2}{2} + y^2 = 1 \Rightarrow c = \sqrt{2 - 1} = 1 \Rightarrow$ foci are $(\pm 1, 0)$; vertices are $\left(\pm\sqrt{2}, 0\right)$

8. $\frac{y^2}{4} - x^2 = 1 \Rightarrow c = \sqrt{4 + 1} = \sqrt{5} \Rightarrow$ foci are $\left(0, \pm\sqrt{5}\right)$; vertices are $(0, \pm 2)$; asymptotes are $y = \pm 2x$

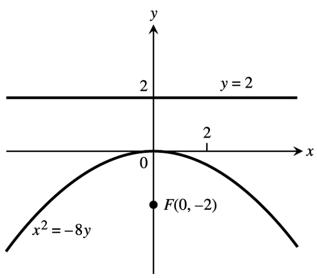
9. $y^2 = 12x \Rightarrow x = \frac{y^2}{12} \Rightarrow 4p = 12 \Rightarrow p = 3$;
focus is $(3, 0)$, directrix is $x = -3$



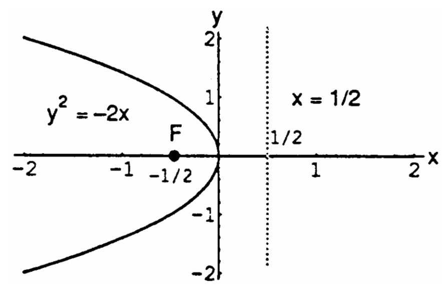
10. $x^2 = 6y \Rightarrow y = \frac{x^2}{6} \Rightarrow 4p = 6 \Rightarrow p = \frac{3}{2}$;
focus is $\left(0, \frac{3}{2}\right)$, directrix is $y = -\frac{3}{2}$



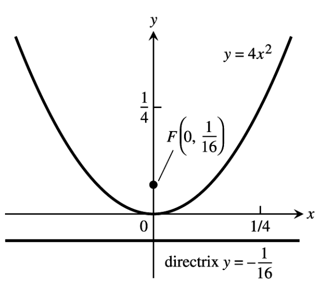
11. $x^2 = -8y \Rightarrow y = \frac{x^2}{-8} \Rightarrow 4p = 8 \Rightarrow p = 2$;
focus is $(0, -2)$, directrix is $y = 2$



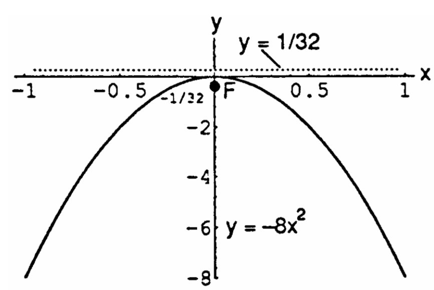
12. $y^2 = -2x \Rightarrow x = \frac{y^2}{-2} \Rightarrow 4p = 2 \Rightarrow p = \frac{1}{2}$;
focus is $\left(-\frac{1}{2}, 0\right)$, directrix is $x = \frac{1}{2}$



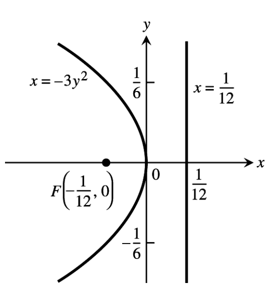
13. $y = 4x^2 \Rightarrow y = \frac{x^2}{\left(\frac{1}{4}\right)} \Rightarrow 4p = \frac{1}{4} \Rightarrow p = \frac{1}{16}$;
focus is $\left(0, \frac{1}{16}\right)$, directrix is $y = -\frac{1}{16}$



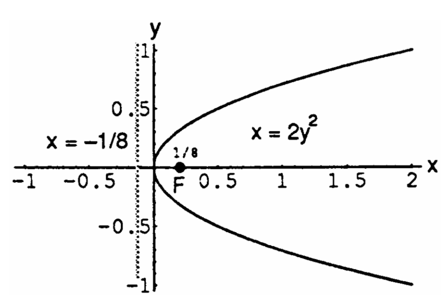
14. $y = -8x^2 \Rightarrow y = -\frac{x^2}{\left(\frac{1}{8}\right)} \Rightarrow 4p = \frac{1}{8} \Rightarrow p = \frac{1}{32}$;
focus is $\left(0, -\frac{1}{32}\right)$, directrix is $y = \frac{1}{32}$



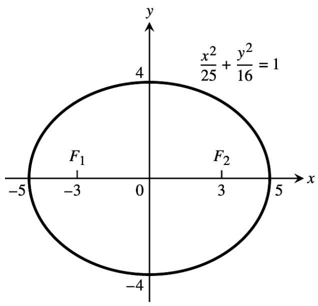
15. $x = -3y^2 \Rightarrow x = -\frac{y^2}{\left(\frac{1}{3}\right)} \Rightarrow 4p = \frac{1}{3} \Rightarrow p = \frac{1}{12}$;
focus is $\left(-\frac{1}{12}, 0\right)$, directrix is $x = \frac{1}{12}$



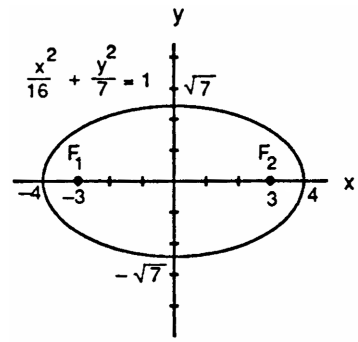
16. $x = 2y^2 \Rightarrow x = \frac{y^2}{\left(\frac{1}{2}\right)} \Rightarrow 4p = \frac{1}{2} \Rightarrow p = \frac{1}{8}$;
focus is $\left(\frac{1}{8}, 0\right)$, directrix is $x = -\frac{1}{8}$



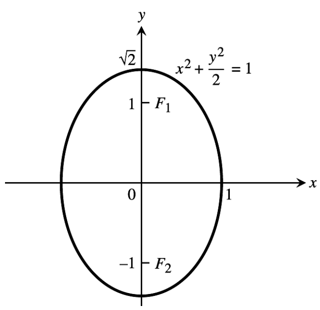
17. $16x^2 + 25y^2 = 400 \Rightarrow \frac{x^2}{25} + \frac{y^2}{16} = 1$
$\Rightarrow c = \sqrt{a^2 - b^2} = \sqrt{25 - 16} = 3$



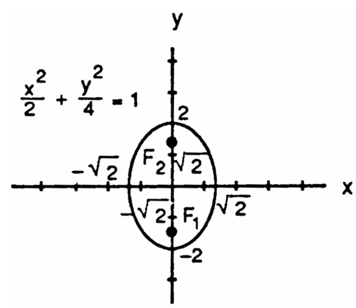
18. $7x^2 + 16y^2 = 112 \Rightarrow \frac{x^2}{16} + \frac{y^2}{7} = 1$
$\Rightarrow c = \sqrt{a^2 - b^2} = \sqrt{16 - 7} = 3$



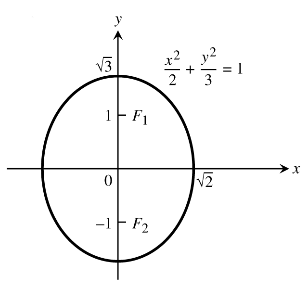
19. $2x^2 + y^2 = 2 \Rightarrow x^2 + \frac{y^2}{2} = 1$
$\Rightarrow c = \sqrt{a^2 - b^2} = \sqrt{2 - 1} = 1$



20. $2x^2 + y^2 = 4 \Rightarrow \frac{x^2}{2} + \frac{y^2}{4} = 1$
$\Rightarrow c = \sqrt{a^2 - b^2} = \sqrt{4 - 2} = \sqrt{2}$



21. $3x^2 + 2y^2 = 6 \Rightarrow \frac{x^2}{2} + \frac{y^2}{3} = 1$
$\Rightarrow c = \sqrt{a^2 - b^2} = \sqrt{3 - 2} = 1$



22. $9x^2 + 10y^2 = 90 \Rightarrow \frac{x^2}{10} + \frac{y^2}{9} = 1$
$\Rightarrow c = \sqrt{a^2 - b^2} = \sqrt{10 - 9} = 1$